

CP Violation in Seesaw Model

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Abstract

We study the structure of CP violating phases in the seesaw model. We find that the 3×6 MNS matrix contains six independent phases, three of which are identified as a Dirac phase and two Majorana phases in the light neutrino sector while the remaining three arise from the mixing of the light neutrinos and heavy neutrinos. We show how to determine these phases from physical observables.

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1 Introduction

The recent experiment on neutrino oscillations [1] strongly suggests the nonzero masses of neutrinos. Among all the models of neutrino masses, one of the most attractive ways to generate the smallness of neutrino masses is the seesaw mechanism [2], which introduces right-handed massive neutrinos. Since the right-handed neutrinos are so heavy, we usually integrate them out and consider the low energy effective theory with only the left-handed neutrinos. However, it is possible that the low energy effective theory might miss some of the essential features of CP violation in the seesaw model. Therefore it is important to study the general structure of the phases in the mixing matrix in the seesaw model which give the CP violation and the lepton number asymmetry.

When we integrate out the heavy mode and consider the low energy effective theory, which we call as “decoupling case” throughout this paper, all the CP violating phases on the lepton sector are contained in the unitary 3×3 MNS mixing matrix [3]. On the other hand, when we consider the full theory of seesaw model with the heavy right-handed neutrinos, which we call as “non-decoupling case”, the MNS matrix is 3×6 matrix. The goal of this paper is to understand degrees of freedom of independent CP violating phases contained in the 3×6 MNS matrix in the non-decoupling case, understand where the difference from the conventional decoupling case arises, and show how the phase including can be determined from experimental observation.

This paper is organized as follows. In section 2, we introduce the extended MNS matrix and count the physical degrees of freedom of the matrix which remains after imposing unitarity conditions and seesaw conditions and using the rephasing symmetry from which we find that there exist six independent phases. As an explicit example, we confirm this result in a special case where the MNS matrix can be explicitly parametrized. In section 3 we study how to determine the phase of the MNS matrix and show that one phase is sensitive in neutrino oscillations, two phases are sensitive in neutrinoless double beta decay and the other three phases are sensitive in lepton number asymmetry. We also present geometrical interpretation of the effects of the phases hidden in Yukawa coupling. Finally, we discuss and conclude the results of our work in section 4.

2 MNS matrix

The Lagrangian of the seesaw model is given as follows

$$\mathcal{L} = -y_{\nu}^{ij} \bar{\psi}_L^i \tilde{\phi} N_R^j - \frac{1}{2} \overline{(N_R^0)^c} m_N N_R^0 - y_e^{ij} \bar{\psi}_L^i \phi e_R^j + h.c., \quad (1)$$

where $\tilde{\phi} = i\tau^2 \phi^*$. Without loss of generality we can start with real diagonal matrix for m_N . This Lagrangian gives mass term as

$$\mathcal{L}_M = -\frac{1}{2} (\bar{\nu}_L^0 \overline{(N_R^0)^c}) M_{\nu} \begin{pmatrix} (\nu_L^0)^c \\ N_R^0 \end{pmatrix} + h.c., \quad (2)$$

where the mass matrix M_{ν} is a symmetric matrix given as follows,

$$M_{\nu} = \begin{pmatrix} 0 & Y_{\nu} \frac{v}{\sqrt{2}} \\ Y_{\nu}^T \frac{v}{\sqrt{2}} & m_N \end{pmatrix}. \quad (3)$$

where we define a matrix as $[Y_{\nu}]_{ij} = y_{\nu}^{ij}$. Hence, this mass matrix can be diagonalized by unitary matrix V , i.e. $m_d = V^T M_{\nu} V$, where V is mixing matrix,

$$\begin{pmatrix} \nu_L^0 \\ (N_R^0)^c \end{pmatrix} = V^* \begin{pmatrix} \nu_L^{\alpha} \end{pmatrix}. \quad (4)$$

The charged current $J^{\dagger\mu} = \bar{l}_L^i V_{MNS}^{i\alpha} \gamma^{\mu} \nu_L^{\alpha}$, where indices $i = 1, 2, 3$ and $\alpha = 1, 2, \dots, 6$. Since we can always start with mass eigenstate for charged lepton, then the MNS matrix is nothing but the mixing matrix V itself,

$$V_{MNS}^{i\alpha} = V^{i\alpha}. \quad (5)$$

It's clear from (5) that V_{MNS} is 3×6 matrix.

We now count the physical degrees of freedom contained in this complex matrix V_{MNS} . For N generations, V_{MNS} is $N \times 2N$ complex matrix. Hence, it has $2N^2$ real and imaginary parts respectively. This matrix satisfies two conditions. First is the unitarity condition which is given in the form below,

$$V_{MNS} V_{MNS}^\dagger = \mathbf{1}_3. \quad (6)$$

$\mathbf{1}_3$ is 3×3 unit matrix. The unitarity condition gives $\frac{N^2+N}{2}$ constraints for real parts and $\frac{N^2-N}{2}$ for imaginary parts, i.e., phases. The second is the very special condition coming from the seesaw type mass matrix. We call it *seesaw condition* from zeros of the matrix (3) and the diagonalization leads to

$$\left(V_{MNS}(m_d) V_{MNS}^T \right)^{ij} = 0. \quad (7)$$

The seesaw condition gives $\frac{N^2+N}{2}$ constraints for real parts and $\frac{N^2+N}{2}$ for phases. By taking into account the unitarity and seesaw conditions, there remain $N^2 - N$ real parts and N^2 phases. Finally, after absorbing N unphysical phases into the charged lepton fields we have $N^2 - N$ independent physical phase. The total independent parameters are summarized in table below.

Table I

V_{MNS}		Constraints			Independent parameter
$[N \times 2N]$		unitarity	Seesaw	Rephasing	
Real	$2N^2$	$N + \frac{N^2-N}{2}$	$\frac{N^2+N}{2}$		$N^2 - N$
Im	$2N^2$	$\frac{N^2-N}{2}$	$\frac{N^2+N}{2}$	N	$N^2 - N$
Total	$4N^2$	N^2	$N^2 + N$	N	$2N(N - 1)$

For $N = 3$, the number of independent phases is 6. The result is different from which could be obtained the one by integrating out the heavy neutrino, i.e. one phase for Dirac neutrino and three phase for Majorana neutrino. We will show how three extra phases come from.

We confirm this result explicitly by taking a specific example. Consider the case where the majorana masses in the heavy right-handed mass has a strong hierarchy in the diagonal basis, namely $m_{N1} \gg m_{N2} \gg m_{N3} (\gg v)$, in which case, the approximate diagonalization of the seesaw matrix is possible [4, 5]. The idea is that starting from the decoupling limit, the mass matrix in Eq.(3) can be diagonalized using a systematic expansion in v/m_N . As a result, the V_{MNS} is given by

$$V_{MNS} = \begin{pmatrix} U & Y_\nu \frac{v}{\sqrt{2}m_N} \end{pmatrix}, \quad (8)$$

where U is a unitary matrix which transforms Y_ν into triangular matrix Y_Δ ,

$$Y_\nu = U Y_\Delta = U \begin{pmatrix} y_1 & 0 & 0 \\ y_{21} & y_2 & 0 \\ y_{31} & y_{32} & y_3 \end{pmatrix}. \quad (9)$$

(See [4] and Appendix A for the proof.) The diagonal elements of Y_Δ are real. The general 3×3 Yukawa coupling has nine phases. The decomposition shows that six of them are included in U while the other three are included in the off-diagonal elements of Y_Δ . Using the decomposition, we can rewrite V_{MNS} as

$$V_{MNS} = U \begin{pmatrix} \mathbf{1}_3, & Y_\Delta \frac{v}{\sqrt{2}m_N} \end{pmatrix}. \quad (10)$$

Three of six phases in U can be absorbed into the definition of charged leptons. Therefore we conclude that V_{MNS} has six phases in total, three of them are in U and the other three are in Y_Δ . This is consistent with the results shown in Table I. The $N \times N$ triangular matrix Y_Δ with real diagonal elements includes $\frac{N^2-N}{2}$ phases. This is exactly the difference of the number of independent phases between non-decoupling case and decoupling case. We call these phases as heavy phase. Then, in general $N(N-1)$ independent phase can be separated as follows

$$N(N-1) = \underbrace{\frac{(N-2)(N-1)}{2}}_{\text{Dirac phase}} + \underbrace{(N-1)}_{\text{Majorana phase}} + \underbrace{\frac{(N^2-N)}{2}}_{\text{heavy}}. \quad (11)$$

Decoupling case

What we call heavy phase in (11) correspond to the phases included in Y_Δ , i.e., the extra phases come from off-diagonal elements of $(Y_\Delta)_{ij}$ for $i > j$. The numbers of CP violating phases for $N = 2$ and $N = 3$ are given in the following

Table II

	Dirac	Majorana	heavy
$N = 2$	0	1	1
$N = 3$	1	2	3

3 CP Violation

Now that we know the number of independent phase of the V_{MNS} matrix, the question is how we can determine these independent phases. To answer this question we discuss the neutrino oscillations, neutrinoless double beta decay and lepton number asymmetry as phenomena which are sensitive to the phases of the extended MNS matrix discussed in the previous sections. This extended MNS matrix which relates ν_L^0 and ν_L as Eq.(4), i.e. $\nu_L^0 = V_{MNS}^{*i\alpha} \nu_L^\alpha$ gives the extended amplitude for both neutrino oscillations and neutrinoless double beta decay.

First, we consider the neutrino oscillations. The transition amplitude $\nu_i \rightarrow \nu_j$ is extended from the decoupling case as follows

$$\begin{aligned} A_{\nu_i \rightarrow \nu_j}(t) = \langle \nu_j | \nu_i(t) \rangle &= \sum_k U_{jk}^* U_{ik} e^{-iE_k t} + \sum_k Y_{jk}^* Y_{ik} \frac{v^2}{2m_{Nk}^2} e^{-iE'_k t} \\ &\simeq \sum_k U_{jk}^* U_{ik} e^{-iE_k t}. \end{aligned} \quad (12)$$

In the last expression, we dropped the contributions from the massive neutrino since they are tiny so that we obtain the same result as in the decoupling case. Hence, we obtain the asymmetry [6]

$$\eta = P_{k \rightarrow l} - P_{k \rightarrow \bar{l}} = 2 \sum_{ij} \text{Im} \left\{ U_{CKM}^{*li}(\theta) U_{CKM}^{lj}(\theta) U_{CKM}^{ki}(\theta) U_{CKM}^{*kj}(\theta) \right\} \sin \left(\delta m_{ij}^2 \frac{L}{2E} \right), \quad (13)$$

since the unitary matrix U can always be transformed $U \rightarrow U_{CKM}(\theta) \text{diag}(1, e^{i\zeta_1}, e^{i\zeta_2})$, where θ is Dirac phase and ζ_1, ζ_2 are Majorana phases (see Appendix A). Eq.(13) represents the CP violation in neutrino oscillations and we can see that neutrino oscillation is only sensitive on Dirac phase living in CKM matrix but not on Majorana phase ζ_1 and ζ_2 .

The similar is also the case in neutrinoless double beta decay. The amplitude is modified from the decoupling case due to the contribution from the heavy neutrino, but after taking into account the scale of momentum-energy and the neutrino masses, the expansion in components may be reduced to the same form as in the decoupling case [7]

$$\begin{aligned}\sum_{\alpha} \frac{m_{\alpha}}{q^2 + m_{\alpha}^2} V_{e\alpha}^2 &= \frac{1}{q^2} \sum_i m_i U_{ei}^2 + \frac{v^2}{2} \sum_i \frac{1}{m_{N_i}^3} Y_{ei}^2 \\ &\simeq \frac{1}{q^2} \sum_i m_i U_{ei}^2(\theta, \zeta_1, \zeta_2).\end{aligned}\quad (14)$$

It is seen that in principle all phases of low energy sector can be determined by using double beta decay experiments, but not for the remaining three phases hidden in Yukawa coupling.

Finally, we consider the lepton number asymmetry in the seesaw model, which was proposed as one of the most promising scenarios of baryogenesis [8]. The lepton number asymmetry has been studied in the flavor basis. Here we rederive the formula for lepton number asymmetry in the mass basis, which is another important result of the present paper. In this basis, the lepton number asymmetry is expressed in terms of only the physical quantities such as mass and V_{MNS} matrix. Therefore in our result the relation of lepton number asymmetry and the CP violating phases is obvious. The relevant part of the Lagrangian which may be derived from Eq.(1) is¹

$$\mathcal{L}_{\chi} = \frac{\sqrt{2}}{v} \chi^+ (\overline{\nu_L^c})^{\alpha} m^{\alpha} (V_{MNS}^{\dagger})^{\alpha k} l_L^k + h.c. \quad (15)$$

From Eq.(15) we get the decay amplitudes \mathcal{M} ,

$$\begin{aligned}\mathcal{M}^{tree} &= \left(\frac{\sqrt{2}m_{\alpha}}{v} \right) (V_{MNS}^{i\alpha})^* (\overline{U}_e L U_N). \\ \mathcal{M}^v &= -i \left(\frac{\sqrt{2}m_{\alpha}}{v} \right) \left(\frac{\sqrt{2}m_{\beta}}{v} \right)^2 V_{MNS}^{j\alpha} (V_{MNS}^{i\beta})^* (V_{MNS}^{j\beta})^* \frac{I(x)}{16\pi} (\overline{U}_e L U_N). \\ \mathcal{M}^s &= -i \left(\frac{\sqrt{2}m_{\alpha}}{v} \right) \left(\frac{\sqrt{2}m_{\beta}}{v} \right)^2 \frac{m_{\alpha}m_{\beta}}{m_{\alpha}^2 - m_{\beta}^2} V_{MNS}^{j\alpha} (V_{MNS}^{j\beta})^* (V_{MNS}^{i\beta})^* \frac{1}{32\pi} (\overline{U}_e L U_N).\end{aligned}\quad (16)$$

where the upper indices show the tree, vertex and self-energy correction respectively, whereas U_e , U_N are spinors of electron and right handed Majorana neutrino, and $I(x)$ is

$$I(x) = \sqrt{x} \left[1 + (1+x) \log \left(\frac{x}{1+x} \right) \right]. \quad (17)$$

with $x = (m_{\beta}^2/m_{\alpha}^2)$. For one-loop amplitudes \mathcal{M}^v and \mathcal{M}^s , we have evaluated the absorptive part only which contributes to the lepton number asymmetry through the interference with tree level amplitude \mathcal{M}^{tree} .

The interference between tree level and one-loop correction of the heavy Majorana neutrino decay gives the asymmetry of the lepton number

$$a(N_{\alpha} \rightarrow l_i^{\mp} \chi^{\pm}) = a_{\alpha i} = \frac{\Gamma(N_{\alpha} \rightarrow l_i^{-} \chi^{+}) - \Gamma(N_{\alpha} \rightarrow l_i^{+} \chi^{-})}{\sum_i \Gamma(N_{\alpha} \rightarrow l_i^{-} \chi^{+}) + \Gamma(N_{\alpha} \rightarrow l_i^{+} \chi^{-})} \quad (18)$$

The contribution from the interference of tree and vertex diagram is given as

¹The process corresponds to $N^{\alpha} \rightarrow l^{\pm} W^{\mp}$. It can be shown that the decay into the longitudinal W mesons can be approximately described by the decay into the unphysical Goldstone boson.

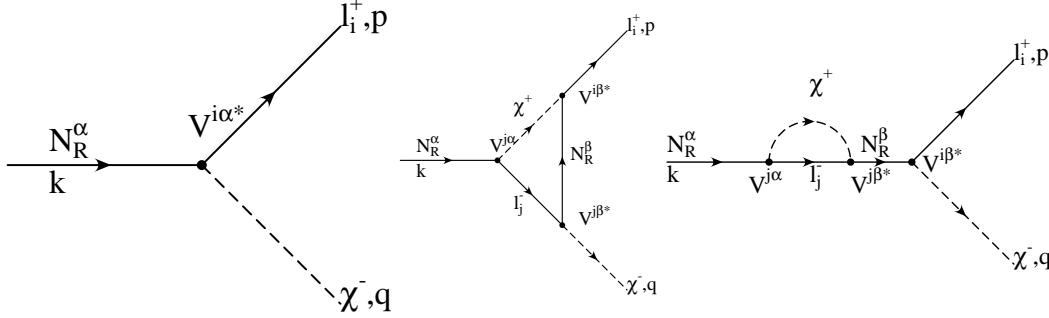


Figure 1: The diagrams of the right-handed neutrino Majorana decay, i.e. tree, vertex and self energy diagram.

$$a_{\alpha i}^v = \sum_{j\beta} \frac{1}{8\pi} \left(\frac{-\sqrt{2}}{v} m_\beta \right)^2 \text{Im} \left[V_{MNS}^{i\beta} V_{MNS}^{j\beta} (V_{MNS}^{j\alpha})^* (V_{MNS}^{i\alpha})^* \right] I \left(\frac{m_\beta^2}{m_\alpha^2} \right) \frac{1}{\sum_i |V_{MNS}^{i\alpha}|^2}. \quad (19)$$

Similarly, the interference between tree and self-energy diagram gives the asymmetry $a_{\alpha i}^s$ as

$$a_{\alpha i}^s = \sum_{j\beta} \frac{1}{16\pi} \left(\frac{-\sqrt{2}}{v} m_\beta \right)^2 \frac{m_\alpha^2}{m_\alpha^2 - m_\beta^2} \text{Im} \left\{ V_{MNS}^{i\beta} V_{MNS}^{j\beta} (V_{MNS}^{j\alpha})^* (V_{MNS}^{i\alpha})^* \right\} \frac{1}{\sum_i |V_{MNS}^{i\alpha}|^2}. \quad (20)$$

The asymmetry in question is summation Eqs.(19) and (20),

$$\begin{aligned} & a(N_\alpha \rightarrow l_i^\mp \chi^\pm) \\ &= \sum_{j\beta} \frac{1}{8\pi} \left(\frac{-\sqrt{2}}{v} m_\beta \right)^2 \text{Im} \left[V_{MNS}^{i\beta} V_{MNS}^{j\beta} (V_{MNS}^{j\alpha})^* (V_{MNS}^{i\alpha})^* \right] \frac{1}{\sum_i |V_{MNS}^{i\alpha}|^2} \\ & \quad \times \left\{ I \left(\frac{m_\beta^2}{m_\alpha^2} \right) + \frac{1}{2} \frac{m_\alpha^2}{m_\alpha^2 - m_\beta^2} \right\}. \end{aligned} \quad (21)$$

From FIG.1, $\alpha = 4, 5, 6$ is index for decaying massive right-handed neutrinos. If we write it with the new index $l \equiv \alpha - 3$, and taking into account the fact that the light neutrinos have mass much smaller than heavy ones as well as the vacuum expectation value of Higgs field v , then the contribution to the asymmetry effectively comes from the heavy part only. Substituting the heavy part of V_{MNS} ; of Eq.(8) into (21) we obtain

$$a_{li} = \frac{1}{8\pi} \sum_{jk} \text{Im} \left[Y^{ik} Y^{jk} (Y^{jl})^* (Y^{il})^* \right] \frac{1}{\sum_i |Y^{il}|^2} \left\{ I \left(\frac{m_{Nk}^2}{m_{Nl}^2} \right) + \frac{1}{2} \frac{m_{Nl}^2}{m_{Nl}^2 - m_{Nk}^2} \right\}. \quad (22)$$

It is clear that the heavy phase appear in lepton number asymmetry at very early of the universe. Summing up to the final charged lepton and expressed in Y_Δ yields

$$a(N_l \rightarrow l^\mp \chi^\pm) = \sum_i a_{li} = \frac{1}{8\pi (Y_\Delta^\dagger Y_\Delta)^{kk}} \sum_k \text{Im} \left[(Y_\Delta^\dagger Y_\Delta)^{lk} \right]^2 \left\{ I \left(\frac{m_{Nk}^2}{m_{Nl}^2} \right) + \frac{1}{2} \frac{m_{Nl}^2}{m_{Nl}^2 - m_{Nk}^2} \right\}. \quad (23)$$

Assuming the off-diagonal elements of Y_Δ are much smaller than the diagonal ones then the explicit form of the asymmetry (23) given as follows

$$a(N_1 \rightarrow l^\mp \chi^\pm) = \frac{-1}{8\pi y_1^2} \left[y_2^2 \text{Im}(y_{21}^2) \left\{ I \left(\frac{m_{N2}^2}{m_{N1}^2} \right) + \frac{1}{2} \frac{m_{N1}^2}{m_{N1}^2 - m_{N2}^2} \right\} \right]$$

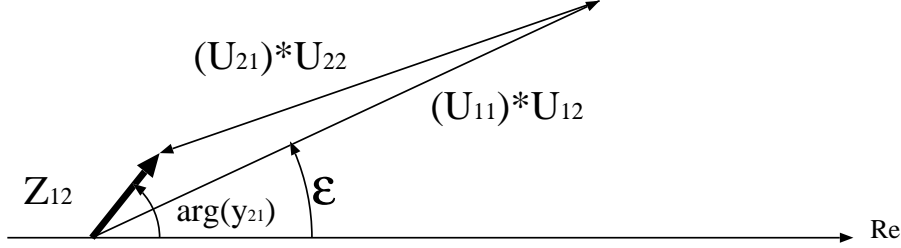


Figure 2: Effect of lepton number asymmetry (2 generation case)

$$+ y_3^2 \text{Im}(y_{31}^2) \left\{ I \left(\frac{m_{N3}^2}{m_{N1}^2} \right) + \frac{1}{2} \frac{m_{N1}^2}{m_{N1}^2 - m_{N3}^2} \right\}. \quad (24)$$

$$\begin{aligned} a(N_2 \rightarrow l^\mp \chi^\pm) &= \frac{1}{8\pi y_2^2} \left[y_2^2 \text{Im}(y_{21}^2) \left\{ I \left(\frac{m_{N1}^2}{m_{N2}^2} \right) + \frac{1}{2} \frac{m_{N2}^2}{m_{N2}^2 - m_{N1}^2} \right\} \right. \\ &\quad \left. - y_3^2 \text{Im}(y_{32}^2) \left\{ I \left(\frac{m_{N3}^2}{m_{N2}^2} \right) + \frac{1}{2} \frac{m_{N2}^2}{m_{N2}^2 - m_{N3}^2} \right\} \right]. \end{aligned} \quad (25)$$

$$\begin{aligned} a(N_3 \rightarrow l^\mp \chi^\pm) &= \frac{1}{8\pi} \left[\text{Im}(y_{31}^2) \left\{ I \left(\frac{m_{N1}^2}{m_{N3}^2} \right) + \frac{1}{2} \frac{m_{N3}^2}{m_{N3}^2 - m_{N1}^2} \right\} \right. \\ &\quad \left. + \text{Im}(y_{32}^2) \left\{ I \left(\frac{m_{N2}^2}{m_{N3}^2} \right) + \frac{1}{2} \frac{m_{N3}^2}{m_{N3}^2 - m_{N2}^2} \right\} \right]. \end{aligned} \quad (26)$$

From the above results we can see that the leptogenesis does not depend on the Dirac and Majorana phases, and depend only on the phase in the off-diagonal Yukawa coupling y_{ij} .

To see more explicitly the effects of Yukawa coupling we describe it in geometrical form below. The effects can be seen from the neutral current $j_{NC}^\mu = \bar{\nu}_L^\alpha \gamma^\mu Z^{\alpha\beta} \nu_L^\beta$, where the explicit form of $Z^{\alpha\beta}$ is given in Appendix B. Tree level Z FCNC in the neutrino sector occurs, and the FCNC comes out at $O(v^2/m_N^2)$. In general, in this geometrical representation, the effect of Z is to open the closed polygon as given in decoupling case by unitarity mixing matrix. We consider two generation case, Z_{12} is given as follows

$$Z_{12} \simeq \frac{m_{\nu_1} m_{\nu_2}}{v^2} \frac{y_{21}}{y_1 y_2^2}. \quad (27)$$

The effect of Z_{12} is to open two lines which are very close together as consequence of unitarity of two generation case and shown in FIG.2.

We consider now the three generation case. The Z_{ij} 's are given

$$\begin{aligned} Z_{12} &\simeq - \frac{m_{\nu_1} m_{\nu_2}}{v^2} \frac{y_{21}}{y_1 y_2^2}, \\ Z_{13} &\simeq - \frac{m_{\nu_1} m_{\nu_3}}{v^2} \frac{y_{31}}{y_1 y_3^2}, \\ Z_{23} &\simeq - \frac{m_{\nu_2} m_{\nu_3}}{v^2} \frac{y_{32}}{y_2 y_3^2}. \end{aligned} \quad (28)$$

In both cases i.e. two and three generation case we have assumed that the diagonal elements of Yukawa matrix are much bigger than the off-diagonal ones. Z_{ij} 's depend on the off-diagonal elements of Y_Δ . Z_{ij} has length and phase, and the effect of this quantity, for example in three generation case, is to open the closed unitarity triangle in decoupling case as shown in FIG.3.

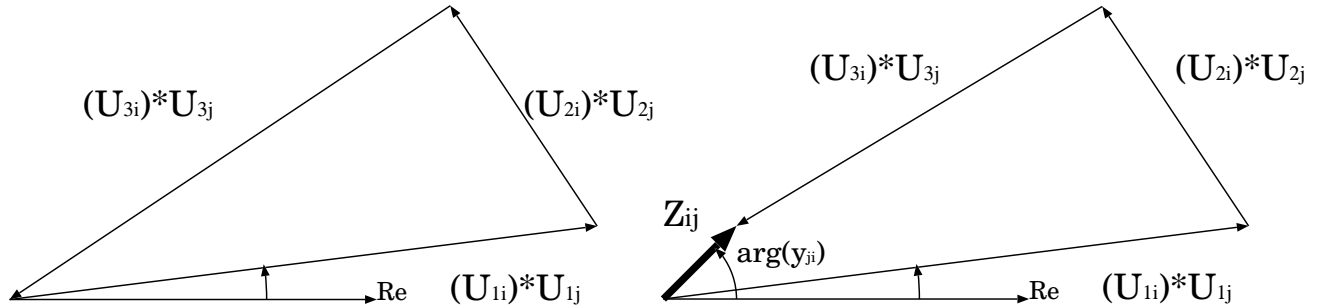


Figure 3: Unitarity “closed” and “open” triangle.

4 Discussion and Conclusion

We studied the structure of CP phase of the seesaw model and found that there are six independent phases rather than three phases as in Majorana neutrino. The extra three phases come from Yukawa coupling between light left-handed neutrino and heavy right-handed neutrino. From the six phases, one, two and three phase are sensitive in neutrino oscillation, neutrinoless double beta decay and lepton number asymmetry respectively. The effect of heavy part hidden in Yukawa coupling -in geometrical representation- open the unitary triangle of low energy sector. Our analysis on the experimental observation of the CP violation and its geometrical interpretation is based on a specific case with hierarchy($m_{N1} \gg m_{N2} \gg m_{N3} \gg v$) where the explicit parameterization by the triangle method is possible. In the present case, we found that the three experimental observations give almost independent information on the CP violating phases. It would be important to see whether this is also the case in more general cases. It would also be interesting to extend our geometrical interpretation to models with sterile fermions where FCNC give non-negligible effects.

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A Triangle method

Here we will show the relation of the phases between the unitary matrix U , Yukawa coupling Y_ν and the triangle form of Yukawa coupling Y_Δ , where the theorem for the relation has been given in [4]. For general Yukawa coupling matrix Y_ν we can always find unitary matrix U which makes it becomes triangular matrix as in Eq.(9). To simplify we write Y_ν in the form

$$Y_\nu = \begin{pmatrix} f_1 & g_1 & z_1 e^{i\alpha_1} \\ f_2 & g_2 & z_2 e^{i\alpha_2} \\ f_3 & g_3 & z_3 e^{i\alpha_3} \end{pmatrix}, \quad (A.1)$$

where f_i and g_i are complex, whereas z_i is real. To do it we have two steps, first introduce a unitary matrix T . Writing Y_ν in the form Eq.(A.1) lead to the form of unitary matrix T

$$T = \begin{pmatrix} u_1 & v_1 & \frac{z_1}{z} e^{i\alpha_1} \\ u_2 & v_2 & \frac{z_2}{z} e^{i\alpha_2} \\ u_3 & v_3 & \frac{z_3}{z} e^{i\alpha_3} \end{pmatrix} \equiv (\mathbf{u} \ \mathbf{v} \ \mathbf{w}), \quad (\text{A.2})$$

where $z = \sqrt{z_1^2 + z_2^2 + z_3^2}$, and the multiplication $T^\dagger Y_\nu$ yields

$$T^\dagger Y_\nu = \begin{pmatrix} \mathbf{u}^\dagger \mathbf{f} & \mathbf{u}^\dagger \mathbf{g} & 0 \\ \mathbf{v}^\dagger \mathbf{f} & \mathbf{v}^\dagger \mathbf{g} & 0 \\ \mathbf{w}^\dagger \mathbf{f} & \mathbf{w}^\dagger \mathbf{g} & z \end{pmatrix}. \quad (\text{A.3})$$

Eq.(A.3) and unitarity condition of T lead to the result for T as follows

$$T = \begin{pmatrix} e^{i\alpha_1} & & \\ & e^{i\alpha_2} & \\ & & e^{i\alpha_3} \end{pmatrix} \begin{pmatrix} -\cos \phi_1 & 0 & \sin \phi_1 \\ \sin \phi_1 \sin \phi_2 & -\cos \phi_2 & \cos \phi_1 \sin \phi_2 \\ \sin \phi_1 \cos \phi_2 & \sin \phi_2 & \cos \phi_1 \cos \phi_2 \end{pmatrix} \begin{pmatrix} e^{i\alpha_4} & & \\ & e^{i\alpha_5} & \\ & & 1 \end{pmatrix}. \quad (\text{A.4})$$

having two angles, (ϕ_1, ϕ_2) and five phases, $(\alpha_1, \dots, \alpha_5)$.

The second step is introducing another unitary matrix S . In this step the present procedure is simpler than the same step in [4]. We do the similar procedure with one in the first step i.e. to make zero the element $[S^\dagger T^\dagger Y_\nu]_{12}$, whereas in the later using Gram-Schmidt diagonalization and then become little complicated. For our requirement we rewrite $T^\dagger Y$ into the form

$$T^\dagger Y_\nu \equiv \begin{pmatrix} a_1 & b_1 e^{i\sigma_1} & 0 \\ a_2 & b_2 e^{i\sigma_2} & 0 \\ \mathbf{w}^\dagger \mathbf{f} & \mathbf{w}^\dagger \mathbf{g} & z \end{pmatrix} = \begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{0} \\ \mathbf{w}^\dagger \mathbf{f} & \mathbf{w}^\dagger \mathbf{g} & z \end{pmatrix}. \quad (\text{A.5})$$

where a_i is complex, and b_i is real. The above form of $T^\dagger Y_\nu$ leads to the form of S

$$S = \begin{pmatrix} \mathbf{s} & \frac{\mathbf{B}}{|\mathbf{B}|} & \mathbf{0} \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} s_{11} & \sin \phi_3 e^{i\sigma_1} & 0 \\ s_{21} & \cos \phi_3 e^{i\sigma_2} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (\text{A.6})$$

where $\phi_3 = \arctan(b_1/b_2)$. The requirement of the form $[S^\dagger T^\dagger Y]_{12}$ is zero and the unitarity of S lead to the form

$$S = \begin{pmatrix} e^{i\sigma_1} & 0 & 0 \\ 0 & e^{i\sigma_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \phi_3 & \sin \phi_3 & 0 \\ -\sin \phi_3 & \cos \phi_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\varepsilon} & & \\ & 1 & \\ & & 1 \end{pmatrix}. \quad (\text{A.7})$$

where ε is a free parameter and can be chosen such that $[S^\dagger(T^\dagger y)]_{11}$ becomes real, and hence, all diagonal elements of $[S^\dagger T^\dagger Y]$ are real. From Eqs.(A.3), (A.4) and (A.7) we obtain U matrix

$$U = TS = \underbrace{\begin{pmatrix} e^{i\alpha_1} & & \\ & e^{i\alpha_2} & \\ & & e^{i\alpha_3} \end{pmatrix}}_{\text{3 phases}} \underbrace{\begin{pmatrix} -c_1 & 0 & s_1 \\ s_1 s_2 & -c_2 & c_1 s_2 \\ s_1 c_2 & s_2 & c_1 c_2 \end{pmatrix}}_{\text{2 angles}} \underbrace{\begin{pmatrix} e^{i\beta} & & \\ & e^{i\gamma} & \\ & & 1 \end{pmatrix}}_{\text{2 phases}} \underbrace{\begin{pmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{1 angle}} \underbrace{\begin{pmatrix} e^{i\varepsilon} & & \\ & 1 & \\ & & 1 \end{pmatrix}}_{\text{1 phase}} \quad (\text{A.8})$$

3 angles + 6 phases

where c_i, s_j are $\cos \phi_i, \sin \phi_j$ and we have reduced four parameters in T and U and expressed as two independent parameters $\alpha_4 + \sigma_1 \rightarrow \beta$ and $\alpha_5 + \sigma_2 \rightarrow \gamma$.

To see the content of independent parameter in Y_ν, U and Y_Δ we consider Eq.(9)

$$Y_\nu = \mathbf{U} \begin{pmatrix} y_1 & 0 & 0 \\ y_{21} & y_2 & 0 \\ y_{31} & y_{32} & y_3 \end{pmatrix}, \quad (A.9)$$

From the above expression we can see that the number of independent parameter of Y is the same with UY_Δ , i.e. 18 parameters. For the phase of the right term, nine parameter is in U and the other nine in Y_Δ . We have seen that the diagonal elements of Y_Δ are real, then this Y_Δ has three independent phases live in off-diagonal elements.

For the physical consideration, the phase α_i of U in Eq.(A.8) may be absorbed by physical fields and U becomes U' ,

$$U' = \begin{pmatrix} -c_1 & 0 & s_1 \\ s_1 s_2 & -c_2 & c_1 s_2 \\ s_1 c_2 & s_2 & c_1 c_2 \end{pmatrix} \begin{pmatrix} e^{i\beta} & & \\ & e^{i\gamma} & \\ & & 1 \end{pmatrix} \begin{pmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\varepsilon} & & \\ & 1 & \\ & & 1 \end{pmatrix} \quad (A.10)$$

After we rearrange the phases and omit irrelevant part we obtain

$$U' = \begin{pmatrix} -c_1 & 0 & s_1 \\ s_1 s_2 & -c_2 & c_1 s_2 \\ s_1 c_2 & s_2 & c_1 c_2 \end{pmatrix} \begin{pmatrix} e^{i\theta} & & \\ & e^{-i\theta} & \\ & & 1 \end{pmatrix} \begin{pmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{-i\zeta_1} & \\ & & e^{-i\zeta_2} \end{pmatrix}, \quad (A.11)$$

where $\theta \rightarrow \frac{\beta-\gamma}{2}$ we identify as Dirac phase, whereas $\zeta_1 \rightarrow \varepsilon$ and $\zeta_2 \rightarrow \frac{\beta+\gamma}{2} + \varepsilon$ as Majorana phases.

B FCNC

Consider the neutral current

$$j_{NC}^\mu = \bar{\nu}_L^{0i} \gamma^\mu \nu_L^{0i} = \bar{\nu}_L^\alpha \gamma^\mu Z^{\alpha\beta} \nu_L^\beta \quad (B.1)$$

where $Z^{\alpha\beta}$ as in [9] is given

$$Z^{\alpha\beta} = (V^\dagger)^{\alpha i} V^{i\beta} = \delta^{\alpha\beta} - (V^{I\alpha})^* V^{I\beta}, \quad (B.2)$$

the explicit form of V is given as follows [5]

$$V = \begin{pmatrix} U & UY_\Delta^* \frac{v}{\sqrt{2}m_N} \\ -\frac{v}{\sqrt{2}m_N} Y_\Delta^T & 1 \end{pmatrix}. \quad (B.3)$$

For $1 \leq \alpha \neq \beta \leq 3$, we obtain

$$Z^{ij} = -(V^{4i})^* V^{4j} - (V^{5i})^* V^{5j} - (V^{6i})^* V^{6j}. \quad (B.4)$$

In non-decoupling case Z^{ij} is not zero then it give effect -for three generation case- on the triangle and becomes open triangle.

Substitute the explicit form Eq.(B.3) into Eq.(B.4) yields

$$-Z_{ij} = \sum_{k=1}^3 (Y_\Delta)^{ik} \frac{v^2}{2m_{Nk}} (Y_\Delta^\dagger)^{kj}, \quad (B.5)$$

and then using relation of left-handed Majorana neutrino mass [4]

$$m_\nu = Y_\Delta \frac{v^2}{\sqrt{2}m_N} Y_\Delta^T, \quad (B.6)$$

we obtain

$$-Z_{ij} = \frac{m_{\nu i} m_{\nu j}}{v^2} \left[(Y_{\Delta}^* Y_{\Delta}^T)^{-1} \right]_{ij}. \quad (B.7)$$

Geometrical representation of this formulation for two generation case is given by the mixing matrix U

$$U = \begin{pmatrix} \cos \theta & -\sin \theta e^{i\varepsilon} \\ \sin \theta e^{-i\varepsilon} & \cos \theta \end{pmatrix}. \quad (B.8)$$

and the triangular Yukawa coupling matrix

$$Y_{\Delta} = \begin{pmatrix} y_1 & 0 \\ y_{21} & y_2 \end{pmatrix}. \quad (B.9)$$

the result is shown by FIG.2. and Eq.(27). For three generation case, the unitarity triangle of U in decoupling case are open by Z'_{ij} s given in Eq.(28) as shown by FIG.3., where Y_{Δ} is given in Eq.(9).

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